**Math 10B with Professor Stankova**
**Worksheet, Discussion #6; Tuesday, 2/12/2019**
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# Algorithms

## Examples

1. Demonstrate bubble sort to sort the list 3, 4, 2, 1.

> **Solution:** On the first pass, we go $3421 \to 3241 \to 3214$, on the next pass we do $3214 \to 2314 \to 2134$ and on the third pass we do $2134 \to 1234$ and on the fourth pass, we make no changes which means the algorithm terminates.

2. Demonstrate the quick sort to sort the list $3, 6, 2, 5, 1, 4$.

> **Solution:** First we place the first number in the appropriate position and put the numbers smaller before it and the numbers larger after while preserving their relative order to get $362514 \to 213654$. Now we do the same on the smaller numbers and the larger to get $21 \to 12$ and $654 \to 546 \to 456$. This finally sorts the list as 123456.

3. Demonstrate the stable matching algorithm when men and women have the preferences $m_1 : w_1 > w_2, m_2 : w_1 > w_2$ and $w_1 : m_1 > m_2, w_2 : m_1 > m_2$.

> **Solution:** Both men will propose to woman 1 and she will choose man number 1. Then man 2 will propose to his next option which is woman 2 and she will accept him. Thus, we get the final pairing $(m_1, w_1), (m_2, w_2)$.

## Problems

4. **TRUE** False The stable matching algorithm with always produce a matching that is stable.

> **Solution:** This question is true if we only apply the stable matching algorithm to situations with two genders. However, this is false in the case of no genders, or the "roommate problem" that Zvezda talked about in class. In the ltter case, applying this algorithm will not always produce a matching that is stable.

5. True **FALSE** There is only one stable matching.

6. Three women A, B, C are proposing to men E, F, G. Their preferences are as follows:

| A | B | C | E | F | G |
|---|---|---|---|---|---|
| $E > G > F$ | $E > G > F$ | $G > E > F$ | $C > A > B$ | $A > B > C$ | $B > C > A$ |

Show the stable matching algorithm with the women proposing to the men by clearly showing all rounds in a table.

**Solution:**

| Men | Rd 1 | Rd 2 | Rd 3 | Rd 4 | Rd 5 |
|---|---|---|---|---|---|
| E | A, ~~B~~ | A | ~~A~~, C | C | $\boxed{\text{C}}$ |
| F | | | | | $\boxed{\text{A}}$ |
| G | C | ~~C~~, B | B | B, ~~A~~ | $\boxed{\text{B}}$ |

7. Sort the list $2, 1, 6, 4, 5, 3$ using both bubble sort and quicksort.

**Solution:** Using bubble sort, we get

$$216453 \to 124536 \to 124356 \to 123456$$

Using quicksort using the last number as a pivot, we get

$$216453 \to 213645 \to (12)3(456).$$